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ABSTRACT

Quasilinear theory is used to calculate the distribution function and neutron rate of a D-T plasma being heated by a lower hybrid wave. It is shown that Q values approaching 1 are possible for sufficiently high bulk temperatures and for small deuterium concentrations.

Lower hybrid wave heating experiments have been carried out on several tokamaks; in several cases the production of fast ion tails due to RF injection were reported.[1–6] Specifically the Alcator A lower hybrid experiment observed the formation of an energetic ion tail in the plasma center having a tail temperature $T_T > 15$ keV and extending out to energies $E > 50$ keV.[4–6] Such ion tails are predicted by the quasilinear theory of Karney.[7] It has been shown that this theory is not inconsistent with the ion tail observations of the Wega experiment.[8] Furthermore, a Monte-Carlo quasilinear ion heating code has shown consistency between its results and the Alcator A RF produced ion tails.[9–11] These results demonstrate a reasonable agreement between experimental results and the quasilinear theory of lower hybrid ion heating.

Stix[12] has employed quasilinear theory to calculate the neutron rates and Q of a D-T plasma being heated by ion cyclotron waves. In this paper we calculate the Q of a D-T plasma being heating by lower hybrid waves employing the quasilinear formalism of Ref. 7. Here we shall show that Q values approaching 1 are achievable with lower hybrid heating by reducing the deuterium fraction and increasing T_e.

Karney[7] has shown that in steady state the ion distribution function can be approximated as $f(\bar{v}) = F_i(v_\perp)\frac{m_i}{2\pi T_i}\exp(-m_i v_\parallel^2/2T_i)$, where

$$F_i(v_\perp) = F_0 \exp\left[-\int_0^{v_\perp} \frac{v_\perp dv_\perp}{v_{ti}^2(1 + D_i(v_\perp)/C_i(v_\perp))}\right] \tag{1}$$

Here we defined[9–11]

$$D_i(v_\perp) = \frac{e^2 E_0^2}{2m_i^2}\frac{\omega^2}{v_\perp^2}\int_{k_{\perp min}}^{k_{\perp max}} \frac{dk_\perp G(k_\perp)}{\Delta k_\perp^2\sqrt{k_\perp^2 v_\perp^2 - \omega^2}}$$

$$C_i(v_\perp) = \sum_\beta \left(\frac{1}{2}\nu_\parallel^{i/\beta} v_\perp^2 + \frac{1}{4}\nu_\perp^{i/\beta} T_i/m_i\right)$$

where $\nu_\parallel^{i/\beta}$ and $\nu_\perp^{i/\beta}$ are defined in Ref. 13, $v_{ti}^2 = T_i/m_i$ and all species temperatures are equal. The summation is over all ion species and electrons, and $\int G(k_\perp)dk_\perp = \Delta = k_{\perp max} - k_{\perp min}$. G $(k_\perp)$ is the spectral shape of $E_0^2(k_\perp)$. For a D-T plasma

$$C_D(v_\perp) = \frac{6\pi n_e e^4 \ln\Omega}{m_D^{\frac{3}{2}} T^{\frac{1}{2}}}\left[\frac{v_{tD}^3}{v_\perp^3}\left(\delta + \frac{2}{3}\left(\frac{1}{2} + \frac{m_D}{m_T}\right)(1-\delta)\right) + \frac{v_{tD}^3}{v_{0D}^3}\right] \tag{2}$$

$$C_T(v_\perp) = \frac{6\pi n_e e^4 \ln\Omega}{m_T^{\frac{3}{2}} T^{\frac{1}{2}}}\left[\frac{v_{tT}^3}{v_\perp^3}\left((1-\delta) + \frac{2}{3}\left(\frac{1}{2} + \frac{m_T}{m_D}\right)\delta\right) + \frac{v_{tT}^3}{v_{0T}^3}\right]$$

$$v_{0D} = 6.99 v_{tD} \qquad v_{0T} = 7.48 v_{tT}$$
$$\delta = n_D/n_e$$

The power dissipated by the wave is[7]

$$P_d = \frac{2\pi n_e}{T}\left[\delta \int_0^\infty \frac{dv_\perp F_D(v_\perp) D_D(v_\perp) v_\perp^3 m_D^2}{1 + D_D(v_\perp)/C_D(v_\perp)} \right.$$
$$\left. + (1-\delta)\int_0^\infty \frac{dv_\perp F_T(v_\perp) D_T(v_\perp) v_\perp^3 m_T^2}{1 + D_T(v_\perp)/C_T(v_\perp)}\right] \tag{3}$$

The neutron rate per unit volume due to tail-bulk collisions is then

$$R_N = n_e^2(1-\delta)\delta \int_0^\infty 2\pi v_\perp dv_\perp \sigma_{DT}(v_\perp) v_\perp (F_D(v_\perp) + F_T(v_\perp)) \tag{4}$$

where $\sigma_{DT}(v_\perp)$ is the neutron production cross section and is a function of the relative ion velocity. We then see that the ratio between neutron and alpha particle production power and the RF power dissipation is $Q = R_N E_N / P_d$, where $E_N = 17.6$ MeV. For $(v_\perp) \gg \omega/k_{\perp max}$, Eq. (1) becomes

$$F_D(v_\perp) \approx F_D\left(\frac{\omega}{k_{\perp max}}\right) \exp\left[-\int_{\omega/k_{\perp max}}^{v_\perp} \frac{dv_\perp (v_\perp/v_{tD}^2)(v_{0D}^3 a_D + v_\perp^3)}{v_{0D}^3 a_D + \gamma_D a_D v_{0D}^3 + v_\perp^3}\right] \tag{5}$$

where

$$a_D = \delta + \frac{2}{3}\left(\frac{1}{2} + \frac{m_D}{m_T}\right)(1-\delta) \sim 1$$
$$\gamma_D = \frac{1}{2}\frac{e^2 E_0^2}{m_D^2}\frac{\omega^2}{k_\perp^3 v_{tD}^3}\frac{1}{C_{D0}}$$
$$C_{D0} = \frac{6\pi n_e e^4 \ln\Lambda}{m_D^{\frac{3}{2}} T^{\frac{1}{2}}} a_D$$

Thus, the shape of the ion tail is strongly characterized by γ_D, and its amplitude is determined by $\omega/k_{\perp max}$.

Figure 1 shows graphs of typical deuterium and tritium ion tails versus $E_\perp$, the perpendicular ion energy for $\gamma_D = 7.05$ and $\gamma_D = 113$. Here parameters similar to those anticipated in the Alcator C lower hybrid heating experiment[14] are used. $k_{\perp max}$ and $k_{\perp min}$ are determined from the dispersion relation

$$k_\perp^4 \epsilon_{xx2} + k_\perp^2 \epsilon_{xx0} + k_z^2 \epsilon_{zz0} = 0 \tag{6}$$

where

$$\epsilon_{xx0} = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \sum_i \frac{4\pi n_e e^2 \delta_i}{m_i \omega^2}$$

$$\epsilon_{zz0} = 1 - \frac{\omega_{pe}^2}{\omega^2}$$

$$\epsilon_{xx2} = -\frac{3}{4}\frac{T}{m_e \omega_e^2}\frac{\omega_{pe}^2}{\omega_{ce}^2} - 3\sum_i \frac{4\pi n_e e^2 \delta_i}{m_i \omega^2}\frac{T}{m_i \omega^2}$$

$$\delta_i = n_i / n_e$$

and where the electric field spectrum is chosen to extend from $k_{zmin} = 2.75\omega/c$ to $k_{zmax} = 3.25\omega/c$. We see that as the γ_D is increased, the extent in energy of the ion tail increases and in this case Q increases from 0.049 to 0.246. We also note that the deuterium tail is generally of greater amplitude than the tritium tail. This ratio of the tritium tail amplitude to the deuterium tail at $v_\perp$ just greater than $\omega/k_{\perp max}$ is

$$\mathrm{RATIO} = \exp\left(-\frac{\omega^2}{2k_{\perp max}^2 T}(m_T - m_D)\right) \tag{7}$$

Nevertheless, the tritium tail will be flatter since its γ_T is larger ($\gamma_i \sim m_i$).

From this expression, we can calculate Q versus γ_D for fixed plasma parameters and fixed wave k_{zmin}, k_{zmax}, and ω. In doing this it is useful to also graph δ_A, where

$$\delta_A = \frac{\Delta V P_d}{A\,S} \tag{8}$$

where ΔV is the volume being heated, A is its external area and S, the power flow into this volume per unit area, is

$$S = \frac{E_0^2 \omega (\epsilon_{xx0} + 2\epsilon_{xx2} k_\perp^2)}{8\pi\, k_\perp} \tag{9}$$

For $\delta_A \sim 1$ the RF power would be absorbed in the order of one pass. For $\delta_A \ll 1$, many passes would be required to absorb the RF power, (which might result in edge plasma heating) whereas for $\delta_A \gg 1$, the RF power would be abosrbed at the edge of ΔV. For $\delta_A \sim 1$ the following calculation of Q is meaningful. (A ray tracing calculation would be necessary to calculate heating details.) For a tokamak $\delta_A = (P_d/S)\Delta r/2$ where Δr is the minor radius of the heating volume.

Figure 2 shows graphs of Q; δ_A and R, the ratio between the RF tail produced neutron rate and the thermal plasma neutron rate for $\delta = 0.5$ and 0.1. We see that higher values of Q are achievable with lower δ. This is easily explainable by noting that both the RF power dissipation and the neutron rate are dominated by the deuterium tail. From Eqs. (1–4), we can then deduce for $\gamma_D \gg 1$

$$Q \approx \frac{(1-\delta)m_D E_n}{6\pi e^4 a_D \ln\Omega} \frac{\int_0^\infty dv_\perp v_\perp F_D(v_\perp) v_\perp \sigma_{DT}(v_\perp)}{\int_{\omega/k_{\perp max}}^\infty dv_\perp F_D(v_\perp)(1+\frac{v_\perp^3}{v_{0D}^3 a_D})} \tag{10}$$

From Eq. (10) we see that Q is not directly dependent on n_e, except that as n_e increases, $k_{\perp max}$ increases (from Eq. (6)), which then increases P_d and lowers Q. Q is proportional to $(1-\delta)$ and will increase as T increases, which increases v_{0D} and lowers the electron drag on the deuterium ions. Lowering n_e sufficiently will raise Q by lowering $k_{\perp max}$; however, as this is done δ_A also rapidly decreases and the absorption becomes too weak for the resulting high Q values to have any real meaning. Also, as γ_D continues to increase Q will decrease, as $\sigma_{DT}(v_\perp)$ decreases at sufficiently large energy while the electron drag term grows. Finally, these results are only useful when $R \gg 1$, as only then is the deuterium tail sufficiently large compared to the thermal tail to make a consideration of an RF Q meaningful.

Figure 3 graphs Q versus δ_D for $T = 5$ keV. We thus see that for larger T, Q can approach 1. While in this case $\delta_A < 1$, it is proportional to Δr; picking a larger heating minor radius would increase it linearly. Figure 3 employs $2.75 < k_z c/\omega < 3.25$; these values of k_z are not strictly appropriate, as they would subject the lower hybrid wave to strong electron Landau damping when $T_e = 5$ keV. Figure 4 shows a similar graph with $2.0 < k_z c/\omega < 2.5$ and with n_e now increased to 1.0×10^{15} cm^{-3}. The previous Q values are approximately recovered; the higher density is only necessary to restore $k_{\perp max} \sim 160$/cm.

In conclusion, we have calculated the lower hybrid wave damping, neutron rates and Q of a D-T plasma. For properly selected n_e, $T > 5$ keV, and small deuterium fraction, values of $Q \sim 1$ can be achieved.

Acknowledgements

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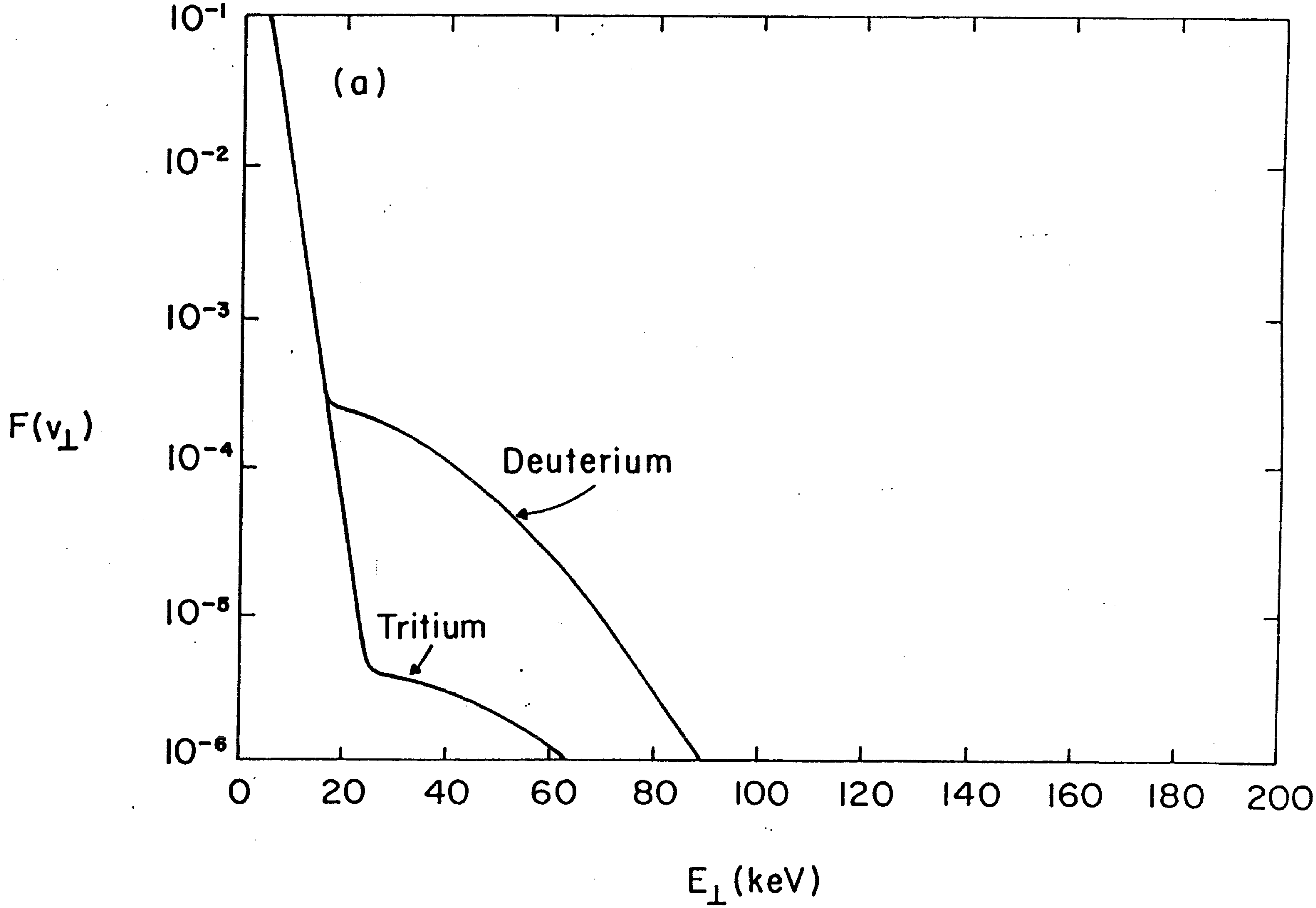
Figure Captions

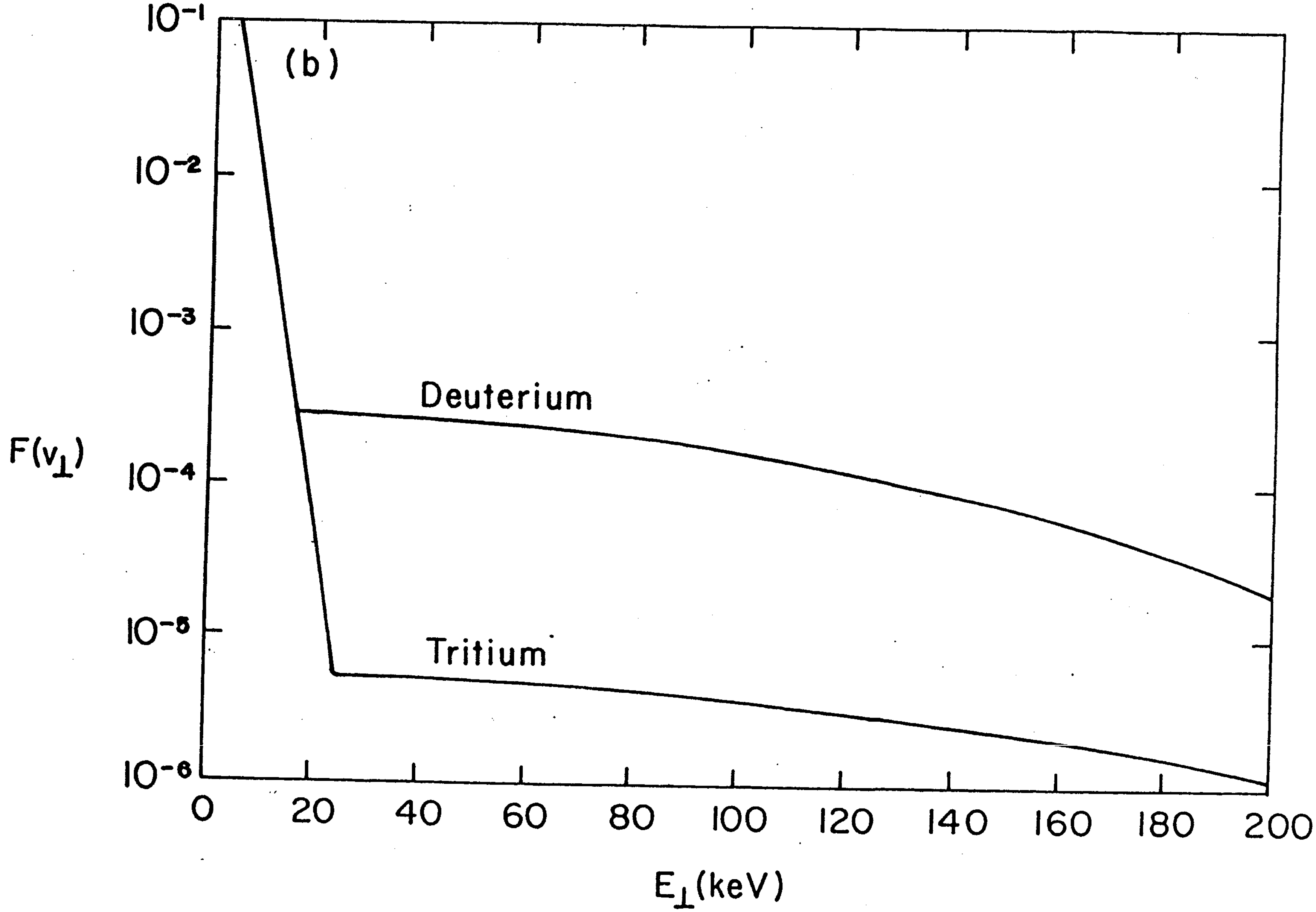
Fig. 1. $F(v_\perp)$ vs. $E_\perp$ for (a) $E_0 = 1$ kV/cm ($\gamma_D = 7.05$) and (b) $E_0 = 4$ kV/cm ($\gamma_D = 113$). Here $f = 4.6$ GHz, $k_{\perp max} = 233$/cm, $k_{\perp min} = 190$/cm, $n_c = 1.0 \times 10^{15}$ cm^{-3}, $T = 2$ keV, and $B_T = 12$ T. In (a) $Q = 0.049$ and in (b) $Q = 0.246$. Here the plasma has equal fractions of deuterium and tritium.

Fig. 2. Q, δ_A, and $\log_{10}(R)$ vs. γ_D for $f = 4.6$ GHz, $T = 2$ keV, $\Delta r = 5$ cm, $k_{zmax} = 3.25\omega/c$, $k_{zmin} = 2.75\omega/c$, and $B_T = 12$ T. In (a) $\delta = 0.5$ and $n_e = 9 \times 10^{14}$ cm^{-3} while in (b) $\delta = 0.1$ and $n_c = 1.2 \times 10^{15}$ cm^{-3}. In both cases n_c has been optimized for the best Q consistent with $\delta_A \sim 1$.

Fig. 3. Q, δ_A and $\log_{10}(R)$ vs. γ_D for $f = 4.6$ GHz, $T = 5$ keV, $\Delta r = 5$ cm, $k_{zmax} = 3.25\omega/c, k_{zmin} = 2.75\omega/c, B_T = 12$ T, $\delta = 0.1$ and $n_e = 6 \times 10^{14}$ cm^{-3}.

Fig. 4. Q, δ_A and $\log_{10}$ (R) vs. γ_D for $f = 4.6$ GHz, $T = 5$ keV, $\Delta r = 5$ cm $k_{zmax} = 2.5\omega/c, k_{zmin} = 2\omega/c, B_T = 12$ T, $\delta = 0.1$ and $n_c = 1.0 \times 10^{15}$ cm^{-3}.


(a)
F(v⊥)
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
Deuterium
Tritium
0
20
40
60
80
100
120
140
160
180
200
E⊥(keV)



(b)
Deuterium
Tritium
F(v⊥)
10⁻¹
10⁻²
10⁻³
10⁻⁴
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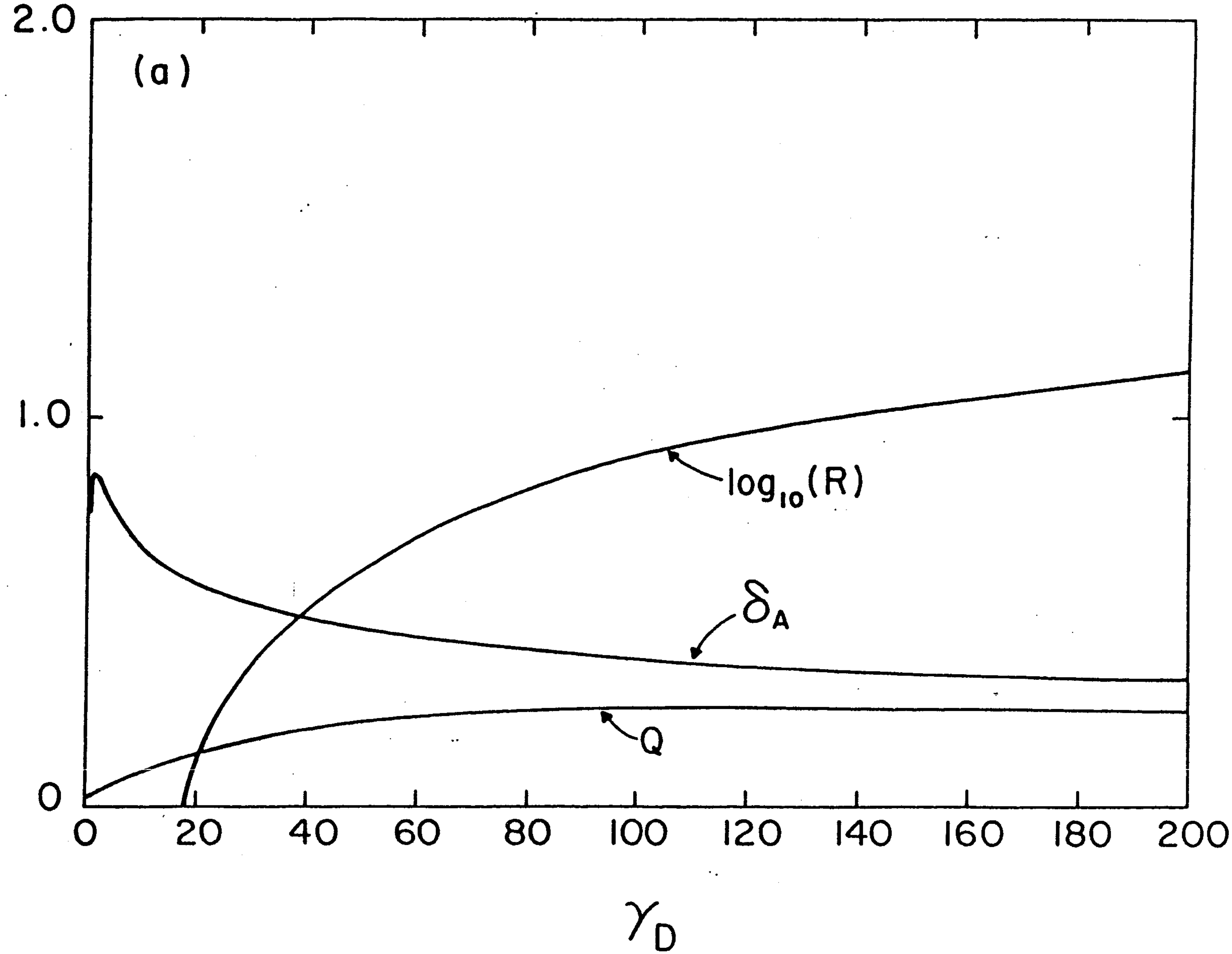
E⊥(keV)



2.0
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(a)
$\log_{10}(R)$
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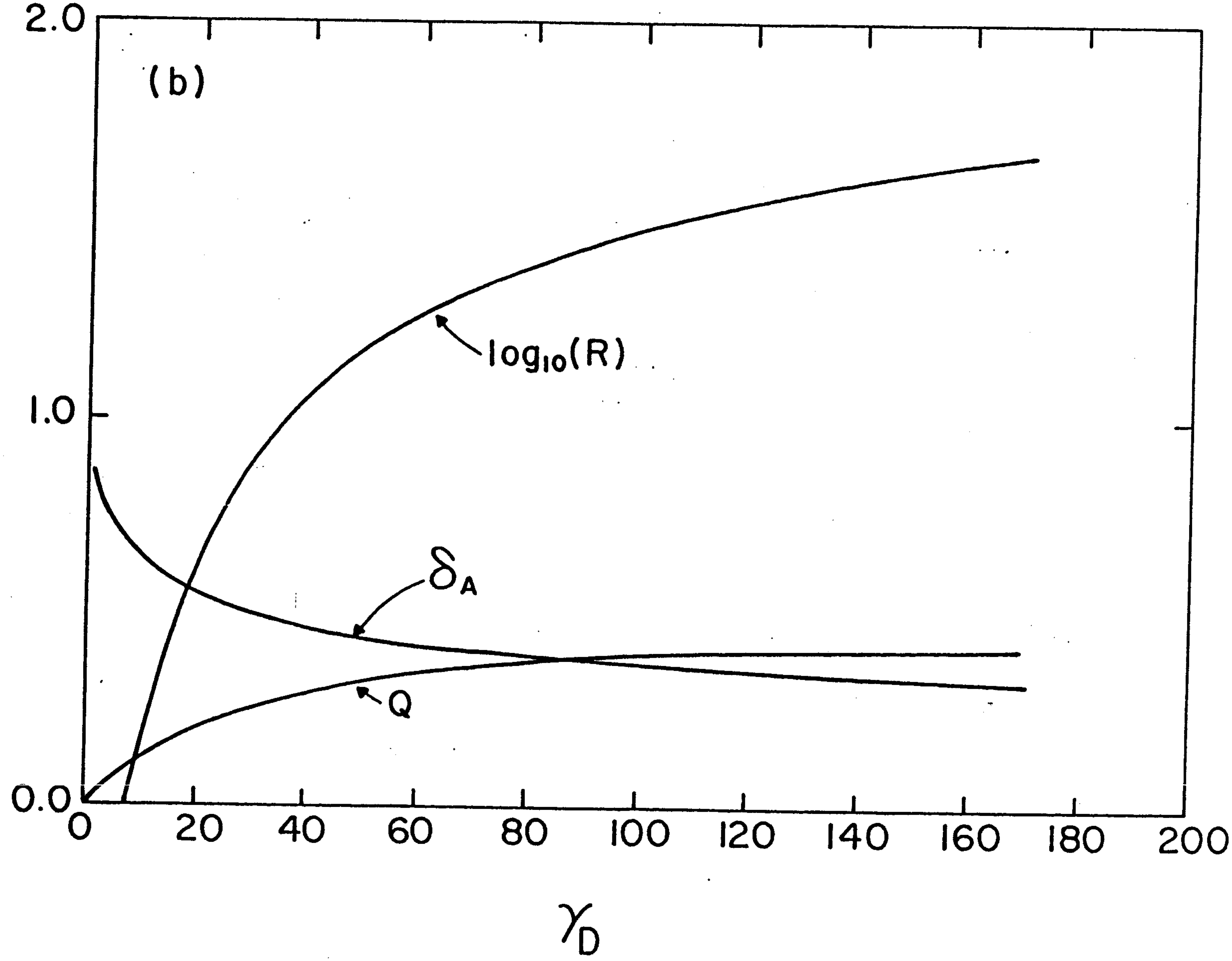
200
γ_D



(b)
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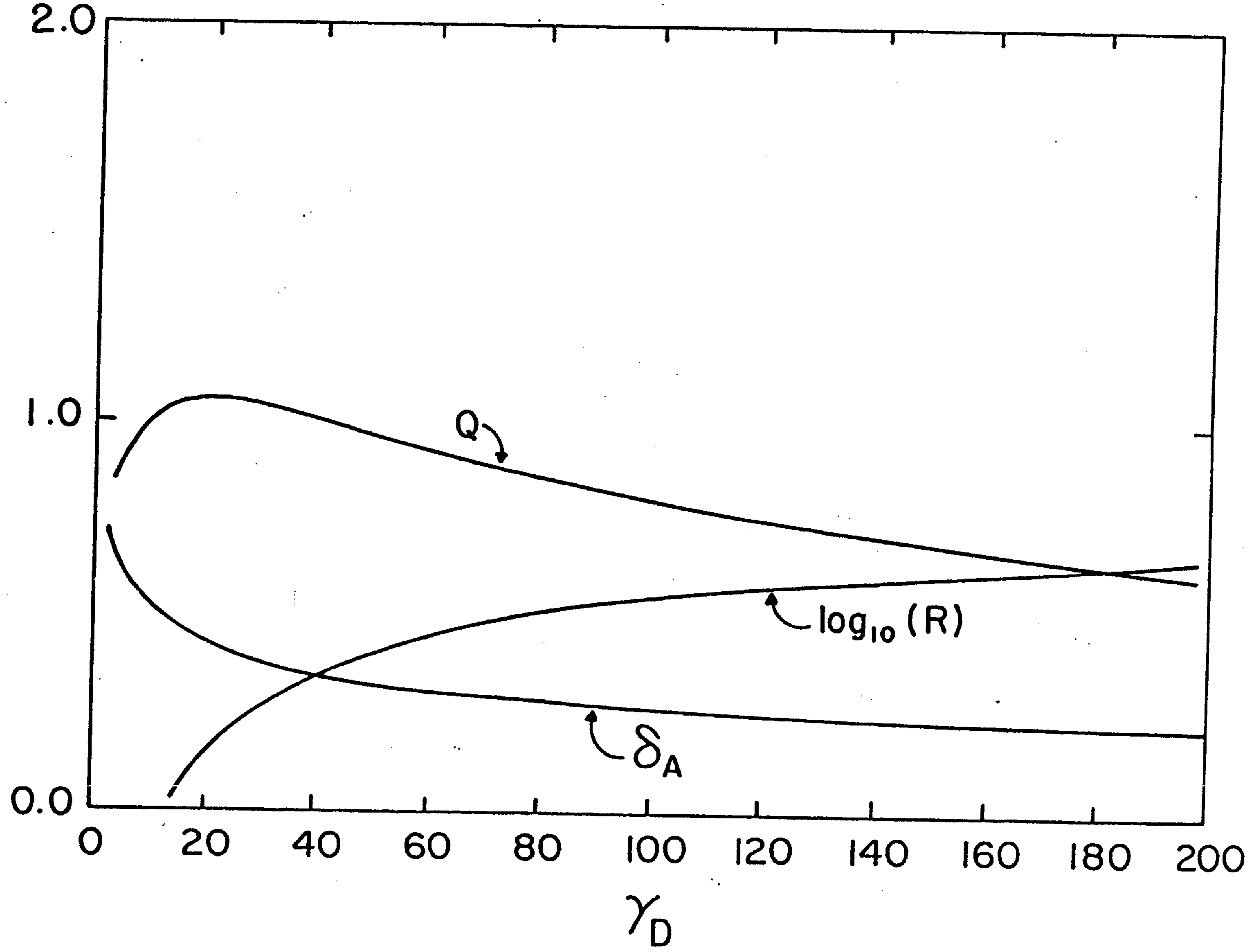
δ_A
Q
γ_D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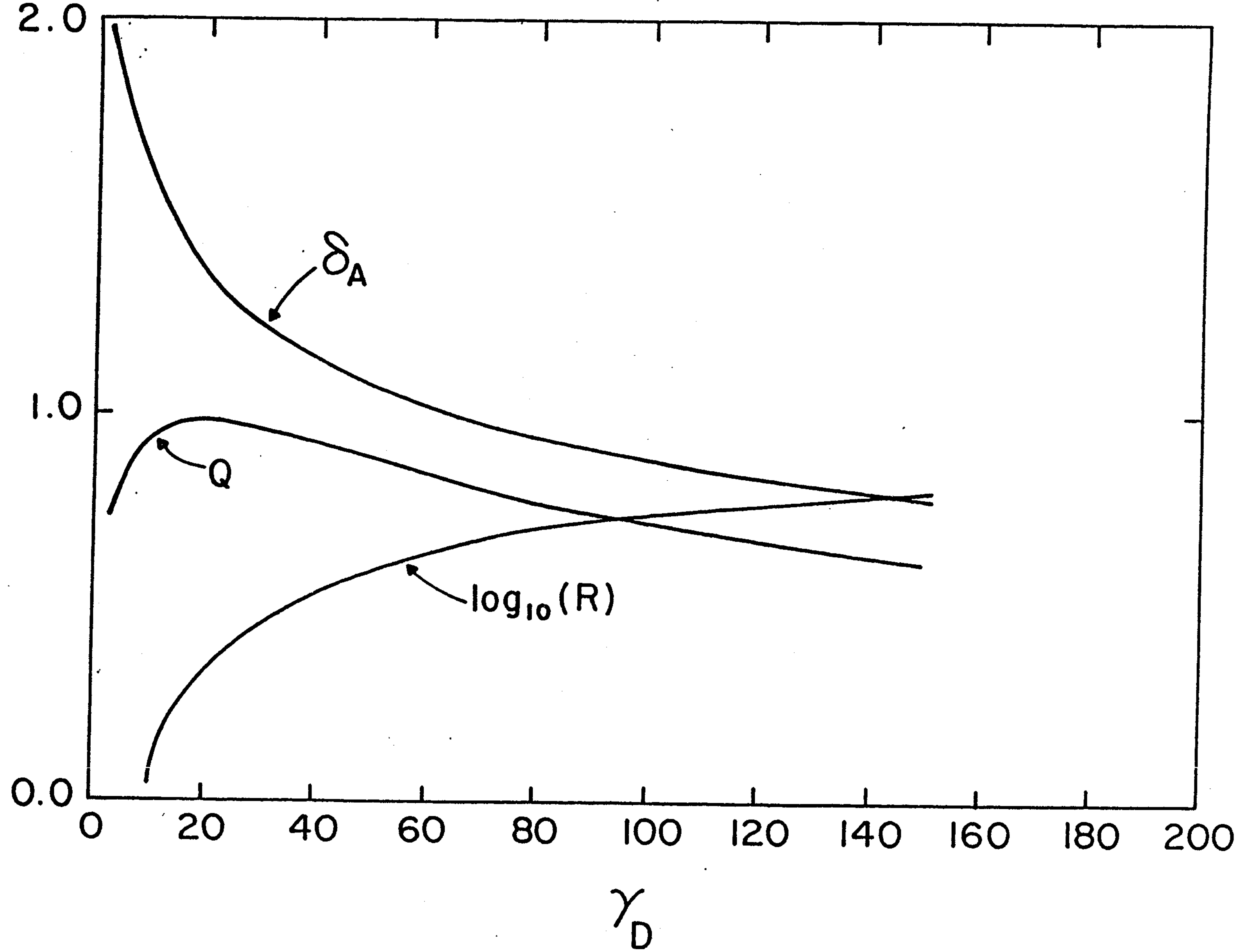
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